

**DEEPER-PENETRATING WAVE APPLICATOR  
IN NONINVASIVE HYPERTHERMIA**

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**ABSTRACT**-The penetration depth of a electromagnetic wave into the lossy medium can be greatly enhanced in the near-field zone of a suitably designed applicator with transient excitation. In the far field of the applicator, EM energy may also decay relatively slowly. We study the decay of EM power in a nondispersive lossy dielectric, and carry out the numerical calculations for the waves in dispersive high water content tissue. The potential for applications of this method is discussed.

### 1. INTRODUCTION

One of the principal obstacles to the use of electromagnetic waves for non-invasive hyperthermia is that the penetration depths of EM waves in lossy biological materials are very small, so that it is difficult to heat cancerous tissue deep in the body. Can we possibly build an applicator which produces a deeper-penetrating wave? In this paper we try to answer this question theoretically. The method proposed here is based upon the concept of the "electromagnetic missile" introduced by T. T. Wu, et al [1], [2]. It was proved that under transient excitation the energy transmitted by an antenna of finite size to a distant receiver in free space can decrease much more slowly than the usual  $R^{-2}$ . Instead, by a suitable choice of excitation, this quantity can decrease as slowly as one wishes, under the physical restriction that the total energy radiated by the antenna is finite. It is the purpose of this paper to determine whether the "electromagnetic missile" effect can be obtained not only in the far-field zone but also in the near-field zone in a lossy, dispersive material. If so, we may hope to use a transient source to obtain a deeper-penetrating wave applicator for noninvasive hyperthermia.

To begin we study the waves in "pure" (nondispersive) lossy dielectric. A transient magnetic current source is used as an excitation. In section 2, a specific formulation is proposed. On the basis of this formulation, it is found in section 3 that for some transient excitations, the waves in nondispersive lossy dielectric

decay slowly. In the far-field zone of the dispersive lossy dielectric, the penetrating EM energy is no longer exponentially decaying, nor does it decay in the form of  $R^{-2}$ . In fact, the decay may be as slow as  $R^{-1}$ . Thus the "electromagnetic missile" can exist in a lossy dielectric. Finally a dispersive lossy material is considered. The dispersion relationship is chosen from the experimentally determined dielectric permittivity and conductivity of various biological tissues. Numerical results for the penetrating EM energy in the dielectric are obtained for four different excitations. The numerical results show that this type of wave applicator will enhance the penetrating of EM energy into lossy biological materials.

### 2. FORMULATION OF THE PROBLEM

Consider a magnetic current source on the circular aperture of a perfect-conducting plane at  $z=0$ , shown in Fig. 1.

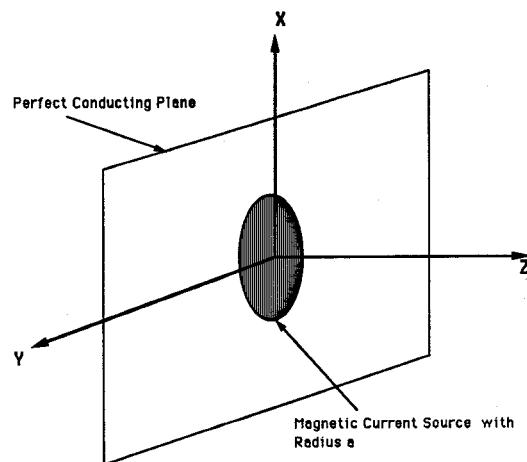


Fig. 1 The configuration of the problem.

In  $z > 0$  there is a semi-infinite lossy dielectric. As in [1], the condition is that the total radiated energy is finite. Let  $P(\omega)$  be the integrated radiated energy per unit frequency at  $\omega$ ; then we require

$$\int_0^\infty P(\omega) d\omega < \infty. \quad (1)$$

The magnetic current distribution is the result of the aperture field at  $z=0$

$$\mathbf{E}_s(\mathbf{r}, t) = \begin{cases} \mathbf{e}_x E_0(x, y, 0) f(t), & \text{if } \rho \leq a; \\ 0, & \text{if } \rho > 0. \end{cases} \quad (2)$$

Let the  $E(\mathbf{r}, \omega)$  be the Fourier transform of  $E(\mathbf{r}, t)$ , viz.,

$$E(\mathbf{r}, \omega) = (2\pi)^{-1} \int_{-\infty}^{\infty} E(\mathbf{r}, t) e^{j\omega t} dt; \quad (3)$$

then, for the distribution (2), at  $z=0$

$$\mathbf{E}_s(\mathbf{r}, \omega) = \begin{cases} \mathbf{e}_x E_0(x, y, 0) \tilde{f}(\omega), & \text{if } \rho \leq a; \\ 0, & \text{if } \rho > 0. \end{cases} \quad (4)$$

Thus the condition (1) can be written

$$\int_0^\infty |\tilde{f}(\omega)|^2 d\omega < \infty. \quad (5)$$

The condition (5) limits the pulse shape of the excitation source. The quantity of central interest is the magnitude of the Poynting vector  $\mathbf{E} \times \mathbf{H}$ , or its time integral

$$F(\varphi, \mathbf{R}) = \int_{-\infty}^{\infty} dt \int_S dS |\mathbf{E}(\mathbf{R}, t) \times \mathbf{H}^*(\mathbf{R}, t) \cdot \hat{\mathbf{n}}|. \quad (6)$$

Here  $S$  is a bounded, oriented surface of finite area away from the antenna, and  $F(\varphi, \mathbf{R})$  is the energy transmitted through  $S$  at  $\mathbf{R}$  in the direction  $\hat{\mathbf{n}}$ . Our goal is that the quantity  $F(\varphi, \mathbf{R})$  decays as slowly as possible in the lossy dielectric. As discussed in [1], equation (6) is also expressed as

$$F(\varphi, \mathbf{R}) = \frac{\pi}{2} \int_0^\infty d\omega \int_S dS |\mathbf{E}(\mathbf{R}, \omega) \times \mathbf{H}^*(\mathbf{R}, \omega) \cdot \hat{\mathbf{n}}|. \quad (7)$$

where  $\mathbf{E}(\mathbf{R}, \omega)$ ,  $\mathbf{H}(\mathbf{R}, \omega)$  are the temporal Fourier transforms. From (4) the magnetic current source,  $\mathbf{m}_s$  is

$$\begin{aligned} \mathbf{m}_s &= \mathbf{E}_s \times \mathbf{n}(\mathbf{r}, \omega) \\ &= \begin{cases} -\mathbf{e}_y E_0(x, y, 0) \tilde{f}(\omega), & \text{if } \rho \leq a; \\ 0, & \text{if } \rho > 0. \end{cases} \quad (8) \end{aligned}$$

Using

$$\begin{aligned} \mathbf{E} &= -\frac{1}{\epsilon} \nabla \times \mathbf{A}_m \\ \mathbf{H} &= -j\omega \mathbf{A}_m - \frac{j\omega}{k^2} \nabla (\nabla \cdot \mathbf{A}_m) \end{aligned}$$

where  $k^2 = \omega^2 \mu \epsilon$  and

$$\mathbf{A}_m = \frac{\epsilon}{4\pi} \int_s \mathbf{m}_s \frac{e^{jkr}}{r} ds'.$$

Here  $r = \sqrt{(x-x')^2 + (y-y')^2 + z^2}$  and  $s$  is the aperture. We consider the EM power into the lossy material along the  $z$ -axis, i.e. on  $x=0$ ,  $y=0$ , and we restrict our analysis to the case where  $E_0$  on the aperture is a constant to obtain  $\mathbf{A}_m$  as

$$\mathbf{A}_m = -\frac{\epsilon}{4\pi} \tilde{f}(\omega) E_0 \mathbf{e}_y \int_s \frac{e^{jkr}}{r} ds' \quad (9)$$

We find

$$E_x(0, 0, z) = \frac{\tilde{f}(\omega) E_0}{2} \left[ e^{j kz} - \frac{z}{\sqrt{a^2 + z^2}} e^{jk\sqrt{a^2 + z^2}} \right]. \quad (10)$$

To find  $\mathbf{H}(0, 0, z)$  we must calculate the term  $\nabla(\nabla \cdot \mathbf{A}_m)$ . Using the identities

$$\frac{\partial}{\partial y} \frac{e^{jkr}}{r} = -\frac{\partial}{\partial y'} \frac{e^{jkr}}{r},$$

and at  $x=0$ ,  $y=0$

$$\frac{\partial^2}{\partial x \partial y} \int_s ds' \frac{e^{jkr}}{r} = 0,$$

we obtain

$$\begin{aligned} \nabla(\nabla \cdot \mathbf{A}_m) &= -\frac{E_0 \epsilon \tilde{f}(\omega)}{4\pi} \mathbf{e}_y \int_s \frac{\partial^2}{\partial y'^2} \frac{e^{jkr}}{r} ds' \\ &= \frac{E_0 \epsilon \tilde{f}(\omega)}{4} \left( -\frac{a^2}{a^2 + z^2} \right) \mathbf{e}_y \left[ -jk + \frac{1}{\sqrt{a^2 + z^2}} \right] e^{jk\sqrt{a^2 + z^2}} \end{aligned} \quad (11)$$

Using (11) and (9) we can obtain  $H_y(0, 0, z)$

$$\begin{aligned} H_y(0, 0, z) &= \\ &\frac{\omega \epsilon E_0 \tilde{f}(\omega)}{4k^2} \left[ -\frac{k(a^2 + 2z^2)}{a^2 + z^2} + \frac{a^2}{(a^2 + z^2)^{3/2}} e^{jk\sqrt{a^2 + z^2}} \right] + \\ &+ \frac{\omega \epsilon E_0 \tilde{f}(\omega)}{2k} e^{jkz}. \end{aligned} \quad (12)$$

Setting  $S$  equal to zero and putting (10), (12) into the expression (7) of  $F(\varphi, \mathbf{R})$  we obtain the formula for the EM power radiated from the source into the dielectric

$$\begin{aligned} F(0, 0, z) &= \\ &+ \int_0^\infty \frac{E_0^2 c}{4} \sqrt{\frac{\epsilon_0}{\mu_0}} \left\{ \left[ e^{jk_0 \sqrt{\epsilon_r} z} - \frac{z}{\sqrt{a^2 + z^2}} e^{jk_0 \sqrt{\epsilon_r} \sqrt{a^2 + z^2}} \right] \right. \\ &\left. + \frac{e^{-jk_0 \sqrt{\epsilon_r} \sqrt{a^2 + z^2}}}{2k_0} \left( -k_0 \sqrt{\epsilon_r} \left( \frac{a^2 + 2z^2}{a^2 + z^2} + \frac{a^2}{(a^2 + z^2)^{3/2}} \right) \right) \right\} \end{aligned}$$

$$+\sqrt{\epsilon_r} e^{-jk_0\sqrt{\epsilon_r} z}]\} |dk_0 \quad (13)$$

where  $k_0 = \frac{\omega}{c}$ ,  $c = \sqrt{\mu_0 \epsilon_0}$ ,  $\epsilon_r$  is the relative complex permittivity of the dielectric outside the source, and  $\tilde{f}(\omega)$  is a function of  $k_0$ . It can be seen that  $\tilde{f}(\omega)$  will contribute to the value of integral in (13). In other words, if we choose different  $\tilde{f}(\omega)$  satisfying the condition (5), we will obtain different EM power in the lossy dielectric at the same distance: The rate of decay of EM power into lossy materials depends on the choice of the pulse shape  $f(t)$ .

### 3. EM POWER IN THE FAR-FIELD ZONE

In order to demonstrate the EM missile effect in the far-field zone we let  $z \gg a$  so  $(a^2 + z^2)^{3/2} \approx z^3(1 + 3a/z)$ , and we consider a nondispersive lossy dielectric outside the antenna. Choosing

$$\tilde{f}(\omega) = \sqrt{\omega/\omega_0} e^{-(\omega/\omega_0)^2}$$

and

$$\sqrt{\epsilon_r} = n_r + jn_i.$$

Then we obtain

$$F(0, 0, z) \approx K_0 \int_0^\infty dk_0 k_0 (c/\omega_0) e^{-k_0^2(c/\omega_0)^2 - 2k_0 n_i z} \{1 - 2 e^{-2k_0 n_i a} \cos(k_0 n_r a) + e^{-2k_0 n_i a}\}. \quad (14)$$

where  $K_0 = E_0^2 \sqrt{\epsilon_0/\mu_0} c/4$ . Using the integration formulas in [3] and applying the conditions of  $z \gg a$  and  $\frac{n_i z \omega}{4c} \gg 1$  to (14), we finally can get an asymptotic formula for the EM power in the far-field zone of a nondispersive lossy dielectric

$$F(0, 0, z) \approx K' \frac{a}{z}, \quad z \gg a, \quad (15)$$

where

$$K' = K_0 \frac{\omega}{32c} |\sqrt{\epsilon_r}|.$$

In order to verify (15), a numerical calculation of relative EM power in the nondispersive lossy dielectric was carried out for  $a=6.0$  cm,  $\epsilon_r = (66, 66)$  and  $\tilde{f}(\omega) = \sqrt{\omega/\omega_0} e^{-\omega^2/\omega_0^2}$  where  $\omega_0 = 0.915$  GHz shown in Fig. 2.

From Fig. 2 it can be seen that the relative EM power for the transient excitation decays much more slowly in the lossy dielectric than for a sinusoidal excitation at  $\omega = 0.915$  GHz. Thus the EM missile effect can be found in a lossy dielectric.

### 4. NEAR FIELD EFFECT

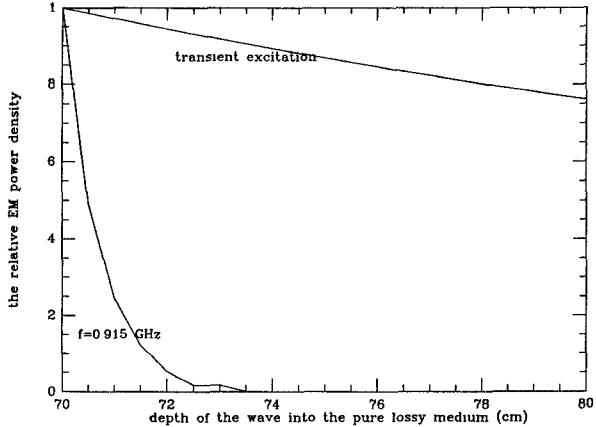


Fig. 2 EM power density in far-field zone of nondispersive lossy dielectric  $\epsilon_r = (66, 66)$ , the radius of the source  $a=6$  cm.

Because of the analytical complication of (13) in the near-field zone we have carried out a numerical calculation using a Fortran program. First, we again considered a nondispersive lossy dielectric with  $\epsilon_r = (66, 66)$  as before. In order to see how the wave penetration into lossy dielectric depended on the choice of pulse shapes we considered four different pulses in our numerical calculation: (a)  $\tilde{f}_1(\omega) = (\frac{\omega}{\omega_0})^{1/2} e^{-4(\frac{\omega}{\omega_0})^2}$ , (b)  $\tilde{f}_2(\omega) = (\frac{\omega}{\omega_0}) e^{-(\frac{\omega}{\omega_0})^2}$ , (c)  $\tilde{f}_3(\omega) = (\frac{\omega}{\omega_0})^{1/2} (e^{\frac{\omega}{\omega_0}} + e^{-\frac{\omega}{\omega_0}})^{-1}$ , (d)  $\tilde{f}_4(\omega) = \delta(\omega - \omega_0)$ . The numerical results are shown in Fig. 3.

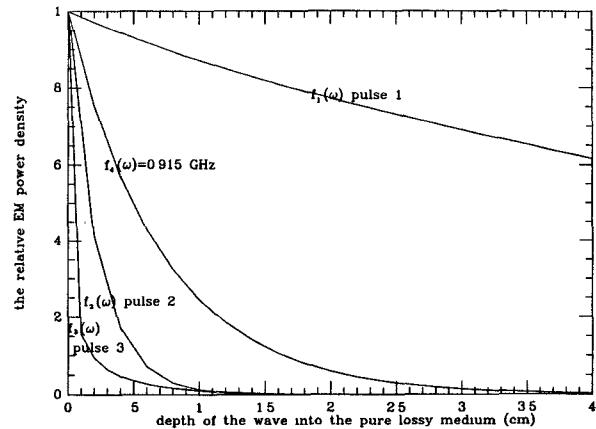


Fig. 3 EM power density in near-field zone of nondispersive lossy dielectric  $\epsilon_r = (66, 66)$ , the radius of the source  $a=6$  cm.

It can be noticed that the choice of the pulse shape  $f(t)$  does affect the penetration of EM energy into lossy material.

Realizing that the assumption of a nondispersive lossy dielectric is not very realistic, we have also taken into consideration the case of dispersive lossy dielectric. For the purpose of application of this method to hyperthermia treatment, the dispersive dielectric was chosen from the reported dielectric permittivity and conductivity of various canine tumor and normal tissues [4]. The frequency response range for the dielectric constants is from 10 Mhz to 18 Ghz, which is sufficient in our calculation for the chosen transient excitation  $\tilde{f}_1(\omega)$  shown in Fig. 4.

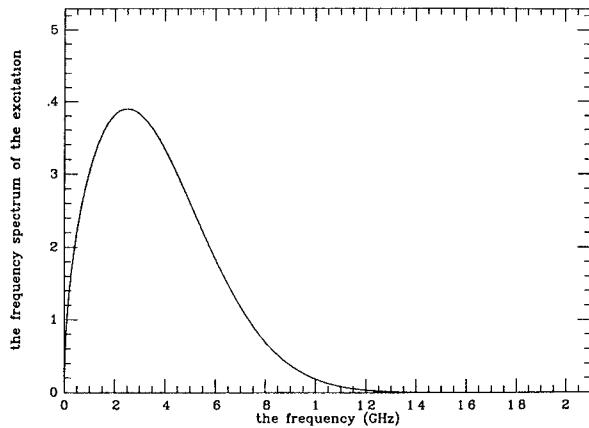


Fig. 4 The frequency spectrum of the transient excitation.

The relative EM powers in the near-field zone and the far-field zone of the dispersive lossy dielectric are calculated and shown in Fig. 5 and Fig. 6 respectively.

It can be seen that the penetration of EM energy into a dispersive lossy material can be enhanced greatly by using the proper transient excitation. In the far-field zone of the material the decaying of the transient excited wave is surprisingly slower than either sinusoidal waves operating at  $f=0.915$  Ghz, or at  $f=0.25$  Ghz which is the "center" frequency for  $\tilde{f}_1(\omega)$ .

## 5. CONCLUSION

It has been found that by a suitable choice of time dependence for the pulse, EM energy transmitted into lossy material can decay very slowly. Although the pulse shape,  $\tilde{f}_1(\omega)$ , we used in the calculation may not be optimal, it does answer the question asked in the section 1 of whether an applicator which produces deeper-penetrating wave is theoretically possible. The idea has applications in noninvasive hyperthermia treatments and may be of potential importance in communication through lossy media.

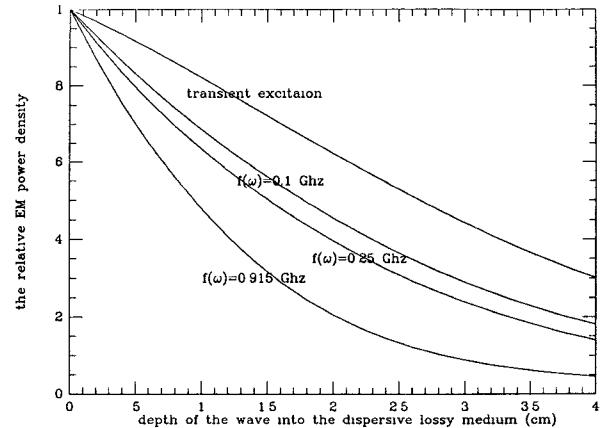


Fig. 5 EM power density in near-field zone of dispersive lossy dielectric, the radius of the source  $a=6$  cm.

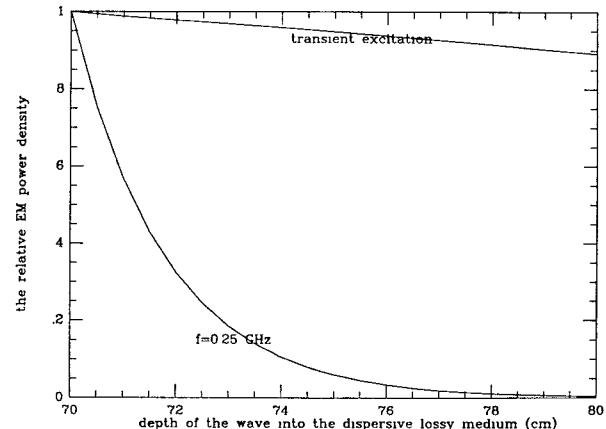


Fig. 6 EM power density in far-field zone of dispersive lossy dielectric, the radius of the source  $a=6$  cm.

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